P425/1

S.6 Pure Mathematics

PAPER 1

JUL-AUG 2025

NN EXAMINATIONS BUREAU

Pure mathematics

PAPER 1

3 hrs.

Instructions to candidates.

- ✓ Answer all the eight questions in section A and any five in section B
- ✓ Any additional question(s) will not be marked
- ✓ All working should be shown clearly
- ✓ Begin each question on a fresh sheet of paper.
- ✓ Silent non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

SECTION A (40 marks)

Attempt all questions in this section

1. Prove by induction that $P_n = n^3 + 2n$ is divided by 3 for non-negative integers of n. (5 marks)
2. Solve for Θ : $3\tan 2\Theta = 2\cos 2\Theta$ for $0^\circ \leq \Theta \leq \pi$ (5 marks)
3. The complex number z is defined by $\frac{z-1}{z+1-2i} = 1 + i$. Express z in form of x + iy. Hence find its modulus. (5 marks)
4. Find the Cartesian equation of the line of intersection of planes 2x – 3y – z = 1 and 3x + 4y + 2z =3. (5 marks)
5. Show that $\int_0^{\frac{\pi}{4}}(2 - cos^2 x)dx = \frac{3\pi-2}{8}$. (5 marks)
6. If y = 2cos(lnx), show that $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (5 marks)
7. Find the equation of the tangent to the curve $\frac{y}{1-y} + \frac{x}{1-x} + 5x - 3y = 0$ at the points (2, 2). (5 marks)
8. Solve the differential equation $x\frac{dy}{dx} = 2x - y$. (5 marks)

SECTION B (60 MARKS)

9. a) Express $10\sin x\cos x + 12\cos 2x$ in the form $R\sin(2x + \alpha)$, hence solve $10\sin x\cos x + 12\cos 2x + 7 = 0$ for $0^\circ \leq x \leq 360^\circ$. (7marks)

   b) Determine the maximum and minimum values of $\frac{3}{10\sin x\cos x + 12\cos 2x - 17}$. State the values of x for which they occur. (5marks)
10. Express $f(x) = \frac{x^3 - x^2 - 3x + 5}{(x-1)(x^2-1)}$ in partial fractions. hence find $\int f(x)dx$ (12marks)
11. a) The sum of the first twenty terms of an A.P is 50, and the sum of the next twenty terms is 200. Find the sum of the first hundred terms of the A.P. (7marks)

    b) In the expansion of $(1 + ax)^4$, the coefficient of x^3 is 1372. Fin the value of a. (7marks)
12. Two lines $r = (3i + 2j - k) + t(2i - 3j + 2k)$ and $L = (-1 + \mu)i + (1 + 2\mu)j + (1 + c)k$. Find the value of c for which the lines intersect and find the coordinates of the point of intersection.

    (12marks).
13. (a) Solve the equation $x^6 - 35x^3 + 216 = 0$ (6marks)

    (b) Given that $t = x - \frac{2}{x}$. Express the following in terms of t

    (i) $x^2 + \frac{4}{x^2}$ (ii) $x^3 - \frac{8}{x^3}$

    Hence solve the equation $2x^4 - 11x^3 + x^2 + 22x + 8 = 0$ (6marks).

14. (a) By sketching, show that an area enclosed by $y = 9 - x^2$ and $y = x^2 - 7$ is $\frac{128\sqrt{2}}{3}$ sq.units. (8marks)

(b). Evaluate $\int_0^\pi \frac{1}{1+x^2} dx$ (4marks)

15. (a) Given that $y = mx + c$, is a tangent to $xy = d^2$, prove that $m = -\frac{c^2}{4d^2}$. (5marks)

(b). Prove that the equation of the tangent to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at the point $(asec\theta, btan\theta)$ is $by + axsin\theta = (x^2 + b^2)tan\theta$. If the normal meets the x-axis p and the y-axis at Q. find the locus of the midpoint of PQ. (7marks)

16. (a). Solve the differential equation; $\frac{dy}{dx} = \frac{y^2-1}{2tan\theta}$. Given that $y = 3$ and $x = \frac{\pi}{2}$ (5marks)

(b) . If $e^x = \cos(x - y)$, show that $\frac{dy}{dx} = \frac{\sqrt{1-e^{2x}} - e^x}{\sqrt{1-e^{2x}}}$ (7marks)

End.

We learn mathematics by doing mathematics.